J. Fluid Mech. (1968), *vol.* 34, *part* 3, *pp.* 595–608
Printed in Great Britain

On the effect of a sharp bend in a fully developed turbulent pipe-flow

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(Received 8 February 1968)

It has been found experimentally that the turbulent pipe flow through a mitred, right-angle bend produces a downstream secondary circulation which does not conform to the twin-circulatory flow usually to be found in pipe bends. The secondary flow is dominated by a single circulation about the axis in either a clockwise or an anticlockwise sense, between which it switches abruptly at a low, random frequency. The phenomenon is explained in terms of the asymmetry of the inner wall separation and the turbulent axial circulation generated in the upstream flow.

1. Introduction

The existence of secondary circulations in curved channel and pipe flows has been known for some hundred years. The first explanation of their production is generally attributed to Thomson (1876), but it is worth noting that Boussinesq (1868) published a paper 8 years earlier, which has generally been overlooked, devoted to approximate solutions of the Navier–Stokes equations for various boundary conditions. He demonstrated that the fully developed, laminar flow through a curved, closed channel of rectangular cross-section consisted of two axial vortices symmetrically disposed about the plane of the bend.

Subsequently, numerous approximate and semi-empirical theoretical solutions have been published both for the laminar and turbulent fully developed flows in curved pipes (e.g. Dean 1927, 1928; Barua 1963; Itō 1959) and in particular for the important practical case of a 90° bend (Hawthorne 1961). These theoretical results are found to be satisfactory for gentle bends but in the majority of applications the ratio d/R (the pipe diameter divided by the mean radius of the bend) is of order unity or greater, and flow separation takes place on the inner wall. This renders accurate analysis intractable, although the trends predicted by the theories still broadly apply.

In addition to the theoretical treatments, detailed experimental investigations have been made into flow in curved channels, especially for circular pipes with a 90° bend and small d/R ratio. Surprisingly little has been done with sharp bends where separation occurs; the most important contribution is that of Weske (1948) who made measurements of the axial and peripheral mean velocity components and the static and total pressure distributions for turbulent flow through a variety of duct and pipe bends. Many of these configurations exhibited separation at their inner wall. Other investigations have been restricted to the measurement of steady state head losses, again for a variety of bends many of which incurred separation. Thus the experimental data on separated pipe-bend flows are limited to the mean quantities despite the essentially unsteady nature of the phenomenon. For this reason the present investigation was made with the intention of determining the time-resolved details of the flow. The primary objective was to establish a qualitative picture of the flow geometry.

As this is the first investigation into this field, fairly severe restrictions were imposed on the scope of the experiment. A 90° bend was used but the geometry was simplified as far as possible by using a mitred intersection ($d/R \rightarrow \infty$) since the resulting salient edge would fix the inner-wall separation point and thus remove one possible degree of freedom. However, it was anticipated and later confirmed that the phenomena described in this paper are not confined to *this* particular limiting geometry. A further restriction on the experiment was to have fully developed turbulent pipe flow upstream of the bend.

2. The experiment

Two sets of apparatus were used. Each consisted of an L-shaped pipe, the mitre bend under investigation occurring at the junction of the two limbs. The more detailed tests were made using air flowing through a $3\frac{1}{2}$ in. bore, smooth-walled tube at a nominal mean speed of 100 ft./sec. The air was sucked through the tube by a compressed-air ejector pump, immediately downstream of a Laval nozzle. This system avoided imposing any swirl to which the flow would inevitably be very sensitive, and also, by running the nozzle choked, fluctuations from the ejector pump were very heavily attentuated. A distance of 33 pipe diameters separated the nozzle from the bend, and a further 45 diameters formed the entry length. The intake was suitably bell-shaped and the first foot of pipe was roughened to increase the rate of growth of the boundary layer. In the vicinity of the bend the pipe was constructed from Perspex to facilitate flow visualization.

The second piece of apparatus was constructed to verify that the rather unexpected observations made in the $3\frac{1}{2}$ in. tube were not peculiar to that particular equipment. A similar L-shaped pipe was fabricated from $1\frac{1}{4}$ in. bore Perspex tube but this time water was used as the working fluid. The entry length of 38 pipe diameters had 4 in. of honeycomb inserted into the inlet to minimize swirl from the reservoir. The maximum velocity obtainable corresponded to a Reynolds number of 4×10^4 based on pipe diameter.

The preliminary measurements made in the air apparatus were intended to confirm the existence of fully developed turbulent pipe flow ahead of the bend. A hot-wire anemometer was used, and immediately it was observed that the flow was unsteady: the indicated mean velocity at a point switched abruptly between two levels at a random frequency of the order of 1 c/s. Since the nozzle was choked, the mean flow could not be time-dependent and thus such a mean-velocity variation implies a two-state, asymmetric profile. The mean-velocity profile averaged over both states was in good agreement with the fully developed profiles of Laufer (1953). The Reynolds number based on the maximum mean velocity was 217,000.

Because of the evidently unsteady nature of the flow, the intake was checked for separation and swirl with smoke, but neither of these phenomena was detected. Thus an unsteadiness associated with the bend was suspected but at a frequency far lower than would reasonably be associated with vortex shedding from the separation at the bend. It was considered advisable to attempt to resolve this complication by flow visualization before making any quantitative measurements.

3. Flow visualization

Flow visualization proved to be difficult because the correct Reynolds number had to be maintained. Reducing the speed to the low value best suited for visualization completely eliminated the unsteadiness. Three different methods were used, each giving some measure of success and confirming the evidence of the others. In the air experiment, which should be regarded as having the more carefully controlled flow conditions, tufts and talc injection were used. For the water experiments, dye injection was the method adopted.

The basic flow pattern was revealed by the talc injection. Sufficiently dense streams of the powder could be injected for several seconds into the flow through holes in the pipe wall by means of a polythene squash bottle. No details of the separated region were discernible but when two injection points were used, located at $\theta = \pm 60°$ (see figure 2) in a plane $\frac{1}{2}$ in. downstream of the inner edge of the bend, the following observations were made. The two talc streams flowed around the wall towards the inside of the bend, passing downstream of what presumably was the separated region. On approaching each other, one became very diffuse whilst the other was entrained by an axial swirling flow and followed a helical path close to the wall of the tube. The initial pitch of the eventually diffused stream was coarser than the other, suggesting a weaker circulation; thus the stronger circulation dominated to produce a nett helical flow downstream. This configuration lasted for periods of typically 1 sec, suddenly switching to the mirror-image state with a nett downstream circulation of opposite sense. On occasions when a particular state lasted for several seconds, it could be seen that the pitch of the helix fluctuated continuously. Blanking off half the intake had no noticeable effect on the frequency of switching or the flow pattern and it was concluded that the phenomena were not a result of intake peculiarities.

Additional information confirming this circulation switching was obtained from wool tufts attached to the pipe downstream of the bend. Three distinct regions of wall flow were distinguished (see figure 1), characterized as follows.

Region (a). The tufts were inclined to the plane of the bend in the downstream direction and switched direction to the mirror-image at a low, apparently random frequency of order 1 c/s. The action of the switching was very fast. The width of the cross-hatching is indicative of the 'intensity' (the magnitude and decisiveness of the change in flow direction).

Region (*b*). Here the tufts did not switch direction through large angle and the flow either side of the plane of symmetry was inclined towards the inside of the bend. The inclination of the tufts did, however, fluctuate at the switching frequency.

Region (*c*). This was essentially a reverse-flow region, extending roughly three-quarters of a diameter downstream, but the tufts were inclined to the same side of the plane of the bend as those in region (*a*).

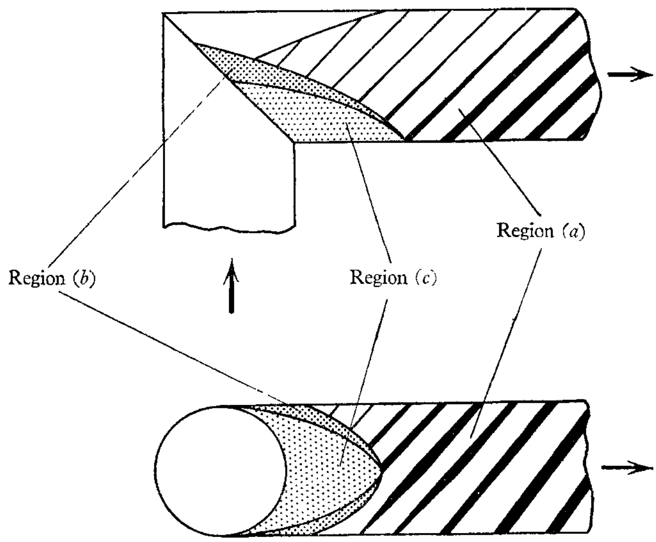

FIGURE 1. Regions of the flow identified by the wool tufts.

On reducing the mean velocity, the flow pattern remained similar down to low velocities (of the order of 10 ft./sec). The randomness of the switching made any accurate estimation of the effect of speed on the frequency difficult; however, at low velocities the switching was slower and much weaker.

The final flow-visualization observations were made with the water model. Dye was injected at various points and, despite diffusion being very rapid, it was evident that a similar 'switching' helical flow was occurring at a comparable, low random frequency. This confirmed that the phenomenon was not peculiar to the air apparatus.

The flow model which was deduced from all the preceding observations in both air and water is shown in figure 2. The important observation was made that the inner-wall separation was not symmetrical but was displaced to one side of the plane of symmetry. It oscillated to either side in sympathy with the switching. Between the inner wall and the flow which separated at the bend there was a strong single cell vortex roughly parallel to the end of the bend. At lower speeds, because of the less rapid diffusion of the dye, it was apparent that at the ends of the separation this vortex trailed downstream. This was more obvious in the arm having the same sense of rotation as the dominant circulation since they reinforced each other. The formation of vortices in the shear layer bounding the separated flow could be seen at the lower velocities.

The flow separated in the outer corner, again enclosing a single-cell vortex with trailing extremities. This vortex also was asymmetric with respect to the plane of the bend and oscillated in sympathy with the switching. In order to assess the importance of the presence of this separation, the outer corner was faired out in both the water and air models until the separation there was suppressed. The downstream flow pattern was virtually unchanged by this modification.

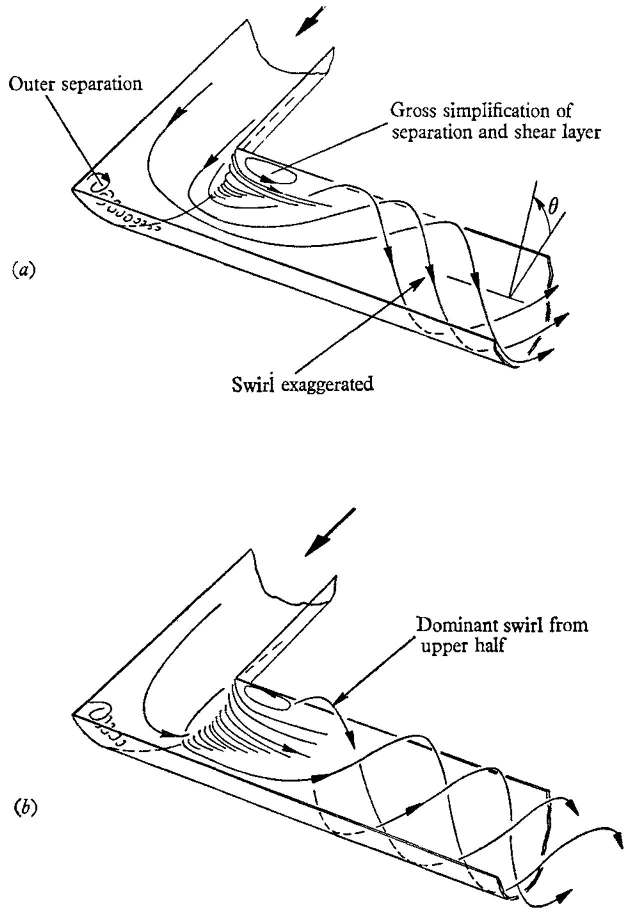

FIGURE 2. The flow pattern observed: (*a*) clockwise sense; (*b*) anticlockwise sense.

Reduction of mean velocity did not fundamentally change the flow pattern until the velocity was low enough for the flow entering the bend to be laminar. The switching then completely ceased and the two classical circulations of opposite sense merely intermingled as they passed downstream.

4. Experimental measurements

It will readily be appreciated that measurements of over-all time-mean quantities in the flow described could not provide information about the instantaneous flow, and therefore could only be of limited value. Despite this objection, in the plane of the symmetry of the bend, the mean velocity field sensed by a hot-wire was least affected by the switching and four down-stream profiles are shown in figure 3. Reverse flow was certainly present near the inner wall in the first two profiles but, in view of the high turbulence intensity and unsteadiness, the hot-wire could not distinguish it, so the profiles are in error in this region. The corresponding turbulence distributions are reproduced in figure 4.

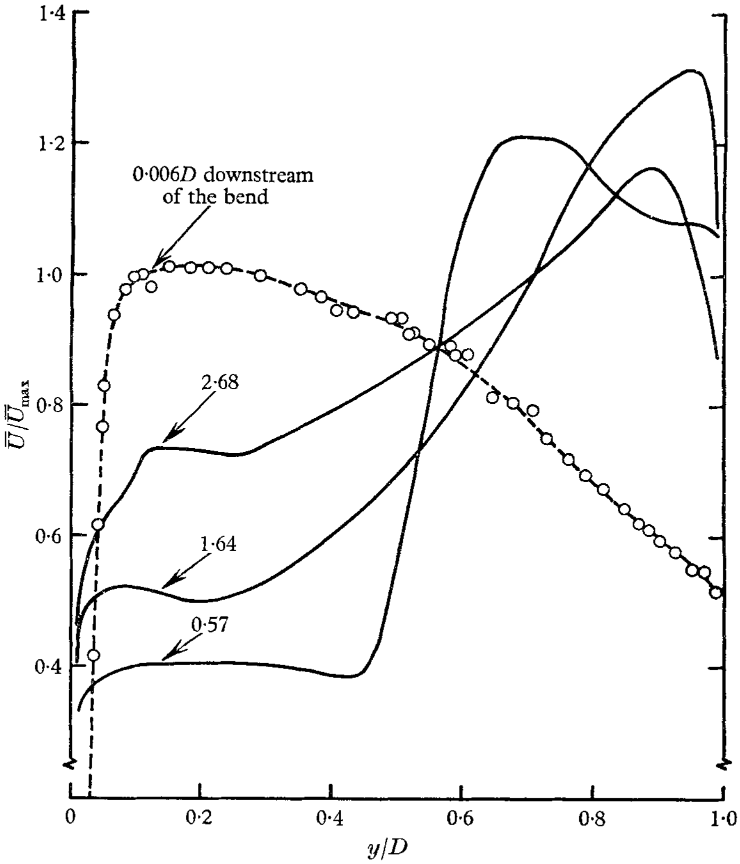

FIGURE 3. Mean axial velocity in the plane of symmetry of the bend.

It was evident that the turbulence intensity at points off the plane of symmetry within roughly three diameters downstream of the bend had two distinct alternative levels, residing for equal periods, on average, at each level and switching in sympathy with the circulation. This suggested that in one state the fluid convected to such a point was 'contaminated' by the separation while in the other state it was not. Thus at two symmetrically disposed points either side of the plane of the bend, states of similar turbulence intensity should correspond to downstream circulations of opposite sense.

To check this prediction and hence confirm the flow pattern, and also to examine in more detail the flow in each state, it was necessary to introduce into the flow a direction-sensitive device. A yawed hot-wire was tried, but although the difference in mean bridge output between the clockwise and anticlockwise circulations was evident, its magnitude was frequently exceeded by turbulent fluctuations.

A simple, crude but successful device was found to be a small gold-shim flag, mounted between two gold-plated contacts against which it was forced alternately by the switching of the circulation. The flag did not switch cleanly but, because of turbulence and contact bounce, produced 'grass' consisting of regular pulses 0·5 ms long with 4 ms spacing. It was evident that while the flow was

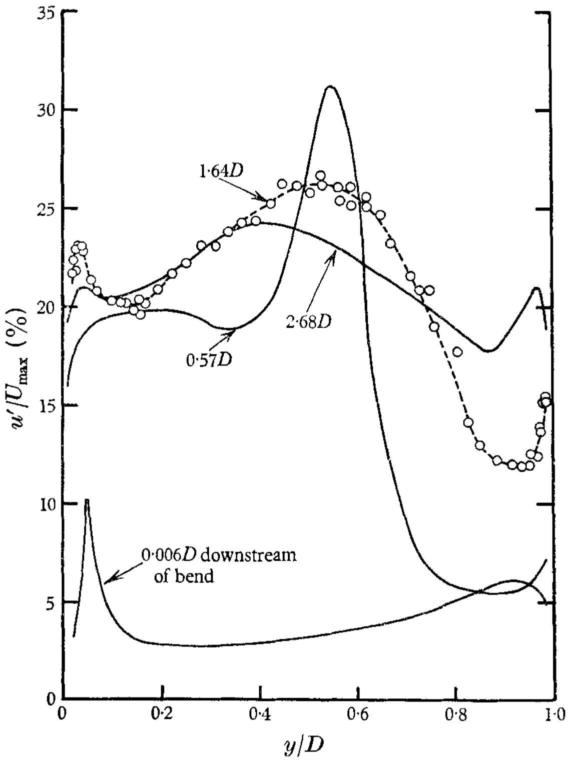

FIGURE 4. Axial turbulence profile in the plane of symmetry of the bend.

predominantly in one state, very brief intervals in the other state frequently occurred lasting for only one or two of the very short pulses. Such switches of between 4–10 pulses were rare. It was considered that these short-duration switches to the other state were due to turbulent fluctuations rather than the change in sense of the secondary circulation, which had a much larger characteristic time scale. They were therefore considered spurious. Further, the response time of the flag was not known. Being a mechanical system it might have lagged the flow considerably, so that any record including a large number of such switches could have been badly in error. It was accordingly decided to ignore switches consisting of less than 4 pulses, and during such switches the circulation was deemed to have remained in its previous state. An electronic logic system was constructed to interpret the pulses from the flag in the desired fashion and to turn on and off the output of a transducer sensing the flow a little upstream of the flag. Thus when the clockwise state of secondary circulation was selected for example, information from that state only was passed on to the processing chain of spectral analyser, vacuo-junction and integrater. The switching function was also used to gate clock pulses into a counter and hence record the total period for which the clockwise state had existed within the run time.

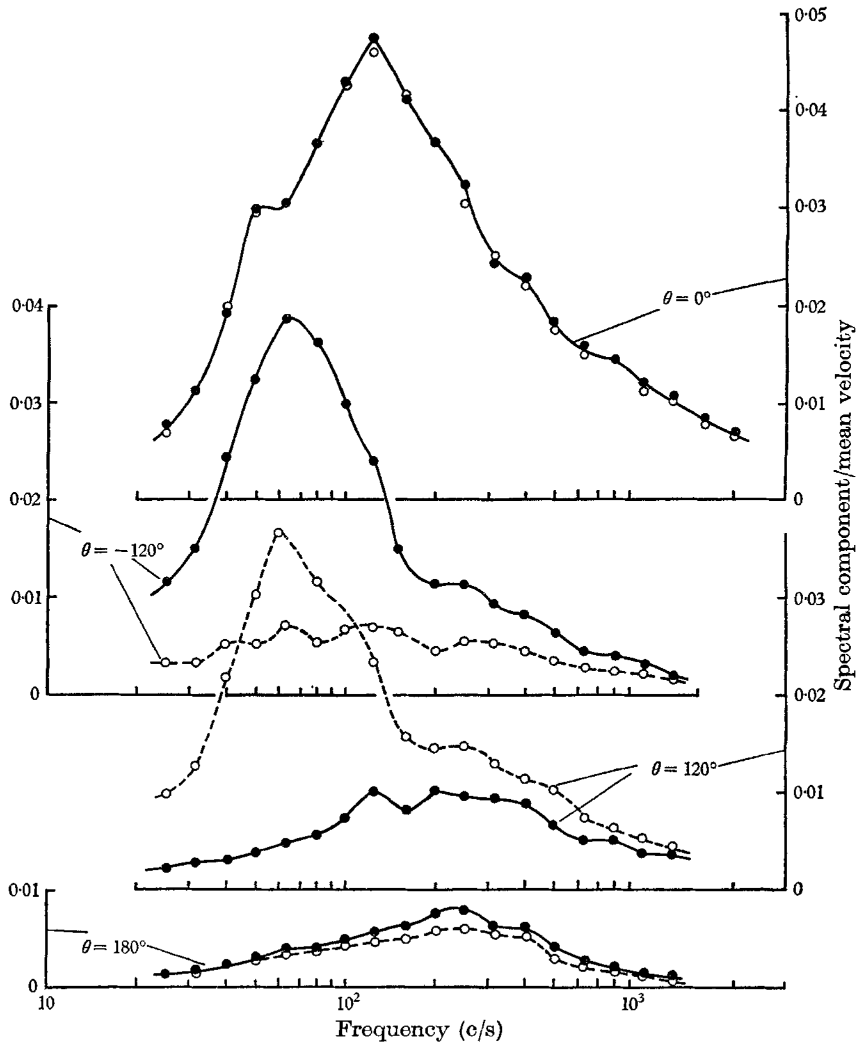

FIGURE 5. Switched turbulent velocity spectra (2·2*D* downstream of bend); ●, clockwise sense; ○, anticlockwise sense.

Axial turbulent velocity spectra obtained by the above methods are shown in figure 5. These were all recorded in the same plane 2·2 diameters downstream of the bend. At $\theta = \pm 120°$ the anticipated similarity of opposite sense spectra is clearly seen, and so is the similarity of the spectra of both senses at the two points on the plane of symmetry (0° and 180°). The high intensity spectra at $\pm 120°$ peak at roughly 65 c/s while the spectra at the inner wall (0°) peak at roughly 120 c/s. Each contains a large component at the peak frequency of the other, but since both spectra are of turbulence derived from the separation it is surprising that they are not more similar. It is suggested that these spectra are derived from different portions of the separation, the lower frequencies associated with the broader central region and the higher frequencies with the smaller extremities of the separation.

Fluctuating wall-pressure spectra of the two states obtained in a similar manner at identical axial and peripheral stations do not display the same separation of intensity levels as the velocity spectra. The r.m.s. readings had two intensity

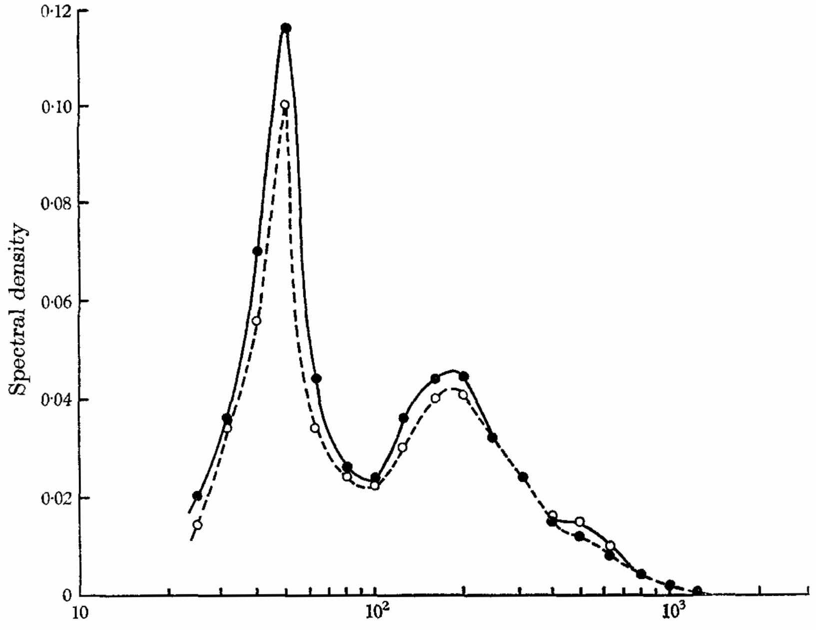

FIGURE 6. Typical switched pressure spectra ($\theta = 120°$): ●, clockwise sense; ○, anticlockwise sense.

levels, switching in sympathy with the rest of the flow, but they did not reside at each level for equal periods as did the velocity record. The lower intensity state was of a more transitory nature. The pressure spectrum was found to be very similar at all stations and in both clockwise and anticlockwise states. This simply reflects the fact that the turbulent velocity is more strongly influenced by local and upstream conditions than the pressure. A typical example of the pressure spectra is shown in figure 6 and is seen to have two peaks related to the peaks of the individual velocity spectra. It is thought that the short periods of low intensity pressure fluctuations observed were caused by the suppression of the large-scale vortices in the shear layer during the re-organization of the separation produced by the switching.

5. The nature of the switching

The mean frequency of the switching was estimated at roughly 1 c/s, at least an order of magnitude below the peak frequency components associated with eddy formation by the separation shear layer. A spectral analysis of the switching frequency, obtained using a pulse length analyser, is shown in figure 7. The mean frequency measured by this method was 0·7 c/s.

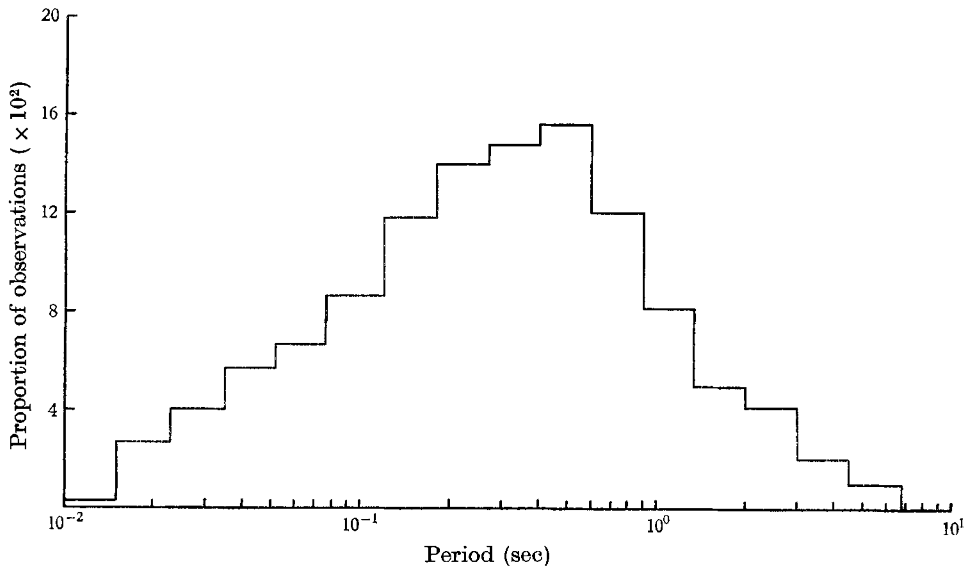

FIGURE 7. Spectrum of switching frequency.

The observations of the rapidity and decisiveness of the actual action of switching suggested that the flow was bi-stable and that a finite and sufficiently large perturbation was required to switch the flow rather than a gradual change in some slowly varying property. The randomness of the switching and the observation that turbulent flow was essential to its occurrence suggested that the perturbation had its origin in the turbulence of the upstream flow.

A possible source of asymmetry in the flow field is the freedom of the inner wall separation to move along the circumference towards one side and so distort the shape of the separated region. This would allow more fluid to flow on paths of higher curvature in one half of the pipe cross-section than the other and would also increase the shear near the inside of the bend in one half and decrease it in the other. From, for example, Hawthorne's (1961) inviscid result for the development of secondary circulation along a streamline, it can be shown that more secondary circulation would then be developed in one half of the section than the other. The reaction of the higher circulation on the separation could cause further departure from symmetry, until eventually a stable configuration was reached. The net circulation further downstream would be the difference between the two circulations close to the corner.

An asymmetrical flow pattern was observed by Weske (1948) who investigated the effect of putting a grid filling half the pipe cross-section ahead of a bend with

d/R ratio $\frac{2}{3}$. When this grid occupied one side of the plane of symmetry, circulation in only one sense was produced downstream of the bend. In the present experiment we are postulating that the non-symmetrical separation is the cause of the single downstream circulation. Unfortunately no further comparison can be made with Weske's results, as when his grid was present there was apparently no separation.

The switching of the circulation to the opposite sense could be brought about by the arrival at the bend of a turbulent pipe-flow fluctuation with sufficient opposing circulation. The circulation required would be just over half the difference between the two circulations immediately after the bend. Measurements of this quantity were not made and would, in fact, be very difficult because of its continual reversal of sign. However, Weske made measurements of the peripheral velocity component. For a 90° bend of $d/R = 1{\cdot}33$ with separation at the inner wall and in the right Reynolds number range, he found the value to be $0{\cdot}18\overline{U}$ ($\overline{U}$ is the average upstream mean velocity). Assuming that this value is appropriate to the present flow, and from the observation that the ratio of the pitches of the two circulations was roughly two, an estimate of the maximum value of the resultant downstream circulation is $0{\cdot}1\overline{U}$. This is consistent with observations made by injecting dye downstream of the bend in the water model. Thus an estimate for the peripheral velocity of a perturbation capable of effecting a switch is $0{\cdot}05\overline{U}$.

The root mean square velocity of circulation about a circuit in a field of isotropic turbulence is given by Townsend (1956) as

$$w' = \left(\frac{L}{\pi r}\right)^{\frac{1}{2}} u'_T \quad (L \ll r),$$

where r is the radius of the circuit, u_T is the turbulent velocity component tangential to the circuit, L is the longitudinal integral scale in the direction of u_T and the primes denote root mean square values. This is viewed as the means by which large eddies arise from a chance orientation of the turbulence field in both isotropic and other types of turbulence. Postulating that this is the means by which the requisite perturbation is generated, an estimate of w' around a circuit whose radius is roughly that of the pipe may be made using the available data on fully developed turbulent pipe flows. Details of the turbulence structure are to be found in Townsend (1956) and Hinze (1959), largely based on the experimental measurements of Laufer (1953). Taking $u'_T/\overline{U} \sim 6\,\%$, $L = 0{\cdot}25a$ and $R \sim a$, $w'/\overline{U}$ is found to be about $1{\cdot}6 \times 10^{-2}$.

Comparing this result with the estimated value of the necessary peripheral velocity, w, of $0{\cdot}05\overline{U}$, it can be seen that a perturbation of roughly three times the standard deviation is required to switch the flow. If the distribution of u_T is Gaussian, then the probability of a deviate lying between three times the standard deviation and infinity is $1{\cdot}35 \times 10^{-3}$. The probability of an eddy of the correct rotational sense to switch the flow will be half this value, and switching will occur at a frequency, n, given by

$$n = Pu/l,$$

where P = probability of a suitable eddy, u = eddy convection velocity and l = eddy axial length scale, which for a turbulent pipe flow is roughly $0{\cdot}8a$. Assuming u is roughly equal to $\overline{U}$, substituting the experimental condition gives the frequency to be 0·6 c/s which is very close to the average frequency observed in the tests (see figure 7).

It must be emphasized that this argument, which is hardly better than an order of magnitude analysis, is not intended to constitute a proof that the mechanism involving the upstream turbulence produces the switching of the secondary flow. In view of the nature of the assumptions such a claim could hardly be justified. However, this analysis does demonstrate that the turbulence could produce sufficiently large perturbations to switch the flow and at the frequency observed. No other satisfactory explanation has been suggested.

6. The effect of variations in d/R and Reynolds number

For the reasons stated in § 1 the majority of the tests were conducted at one speed and with the sharp mitred bend ($d/R \to \infty$). However, some additional measurements were made varying these parameters.

First, the influence of the inner-wall bend radius was investigated by replacing the mitred bend by a series of bends with radiused inner corners. The largest radius used gave an effective d/R ratio of unity, thus overlapping the case tested by Weske where d/R was equal to 1·33, previously referred to. Details of these experiments are to be published elsewhere, but one important observation was that in every case the switching as described here occurred.

This immediately poses the question of why the switching was not observed in Weske's flow, which had the essentials of an inner wall separation and upstream turbulence. It is, however, possible that Weske overlooked the phenomenon for the reason that his measurements of peripheral velocity were made in the outlet plane of the bend, where the flow is separated at the inner wall. Considering the equivalent plane in the case of the mitre bend, it will be recalled that the wall peripheral velocity component was not greatly affected by the switching, being the inward circulation region (*b*) of figure 1. Further, in this plane the circulations would appear roughly symmetrical and the downstream flow could then be assumed identical to the well known secondary flow in smooth bends. Fluctuations and deformation of the separation would not necessarily be obvious, either because of the high turbulence levels or because of the damping in the manometer system: no flow visualization was performed.

When consideration is given to the influence of Reynolds number on the switching frequency no simple prediction can be made, for although the turbulent circulation velocity, $w'/\overline{U}$, is unlikely to change significantly (Laufer's results show a variation of only a few per cent in u'_T in the region of interest for an order of magnitude change in Re), the effect on the downstream circulation is not readily assessed. With the air apparatus it was possible to vary the mean velocity only over a limited range but even so, as reference to figure 8 will show, the Strouhal number derived from the switching of the flag shows a considerable change with velocity. Measurement below 20 ft./sec was not possible because of the diminishing force acting on the flag and the weakening of the switching. Making the assumptions of the previous section, the value of the Strouhal number can be derived as

$$\frac{2na}{\overline{U}} = \frac{1 \cdot 25}{\sqrt{(2\pi)}} \int_p^\infty e^{-\frac{1}{2}\sigma} d\sigma,$$

where $p = w/w'$.

A reasonable fit to the experimental results (see figure 8) is given by assuming p to vary as $\overline{U}^{-0 \cdot 2}$ which implies that the shear varies as $\overline{U}^{0 \cdot 8}$.

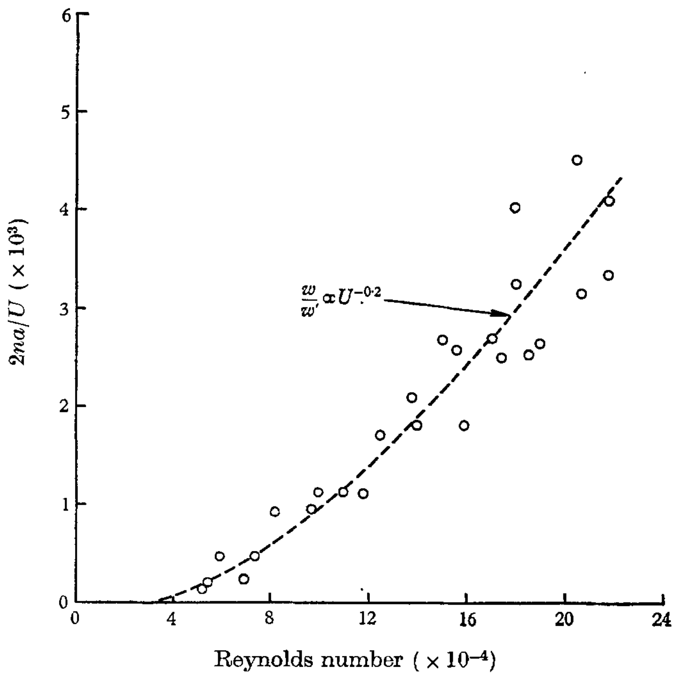

FIGURE 8. The effect of speed on the switching frequency.

7. Conclusion

The flow through sharp pipe-bends is seen to differ from the classical twin-vortex secondary flow and to be an essentially unsteady, bi-stable flow.

The two stable configurations, which are mirror images of each other, are characterized by the inner-corner separation region being displaced into an asymmetric position thus biasing the main flow into the other half of the bend. This produces a swirling flow which sweeps around close to the walls of the downstream leg of the pipe. The core of this swirling flow appears to entrain the eddies shed from the shear layer bounding the flow separation from the inner wall. The necessary conditions for the switching to occur appear to be that the bend must be acute enough to cause separation at the inner wall, and that the flow entering the bend must be of high Reynolds number and probably turbulent. The shape of the cross-section is also a relevant parameter, since no evidence of switching was found by Curtiss, Feil & Liquorrik (1964), in a rectangular section duct bend where the aforementioned conditions were satisfied and flow visualization was employed. Thus the top and bottom walls of such a duct can restrain the separation.

In this experiment with fully developed turbulent flow, spontaneous abrupt changes from one bi-stable state to the other occur randomly and at a very much lower characteristic frequency than would be usually associated with eddy-shedding from the separation region. It is proposed that the switching is the result of occasional existence of turbulent circulation entering the bend with a magnitude opposite to and slightly larger than half the downstream circulation. Rotation in the reverse direction is thus established and the flow then settles into the other stable mode.

As a subsidiary verification of this explanation, a simple modification of the apparatus was made by adding swirl vanes to the intake. The anticipated biasing of the secondary flow to the corresponding state was observed although occasional transitory switches to the opposite state still occurred.

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